What Comes Before the Standard Algorithm?

59 + 13

- 3rd Grade 59+13

Caleb went on a hike in Death Valley. He took a tram up the mountain and started at an elevation of _____ feet above sea level. He hiked down in elevation _____ feet until he got back to the car. At what elevation was the car?

(90, 54) (900, 547) (1020, 547) (1020, 1547)

90-54
90-50=40
40-4=36 feet above sea level

4th Grade

4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.

3rd Grade

2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

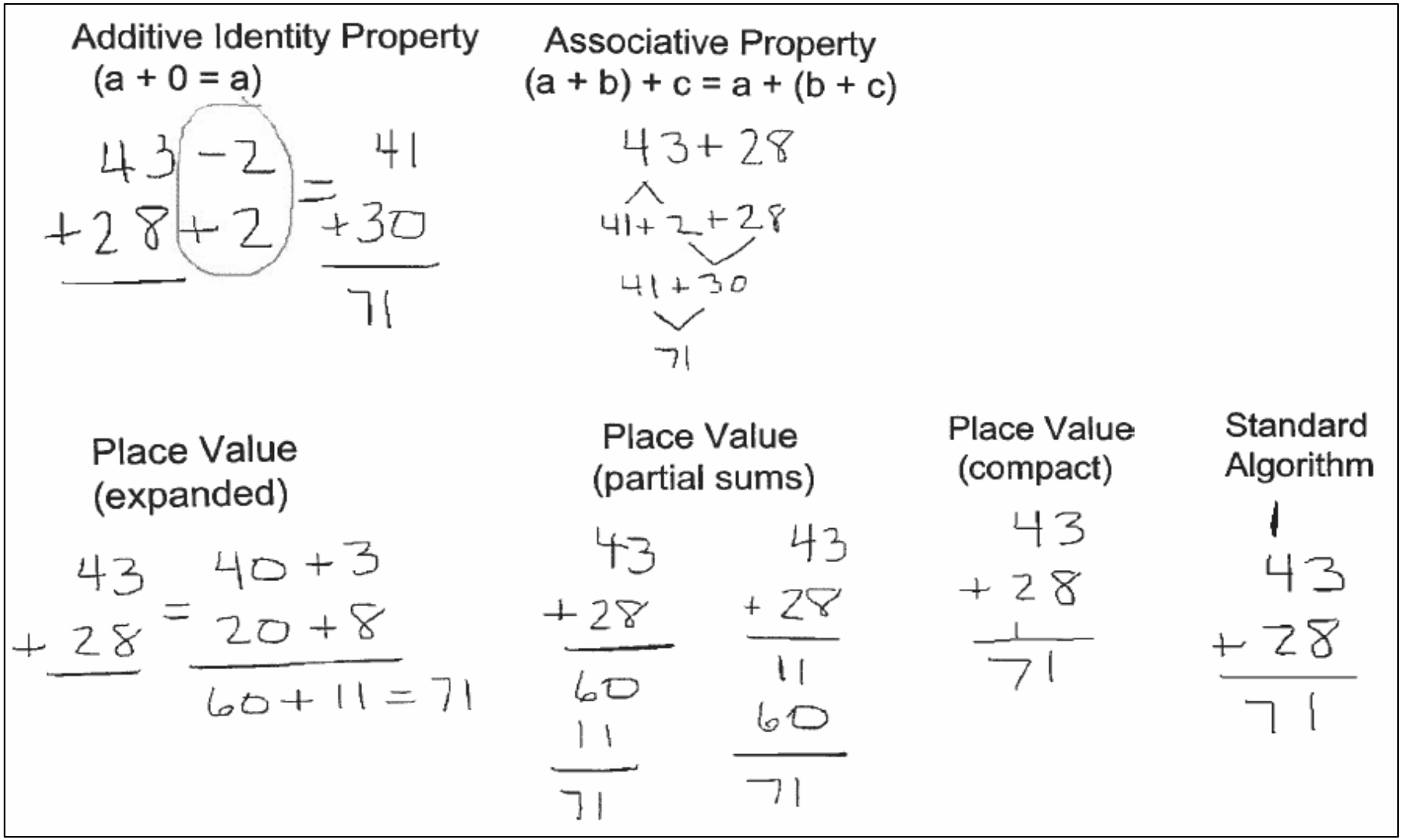
2nd Grade

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

7. Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Strategies and Algorithms

Properties of Operations, Place Value, Relationship btwn + & -



Use place value and properties of operations to add and subtract

- Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction;
- relate the strategy to a written method.

Caleb rode the tram _____ meters up the mountain and then hiked up _____ more meters. How far will he need to hike to get back down to the tram station?

(37, 24) (317, 264) (387, 264)

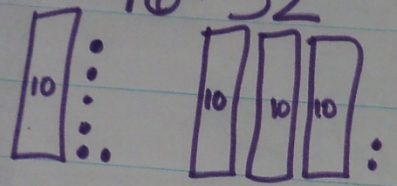
Caleb rode the tram _____ meters up the mountain and then hiked up _____ more meters. How far will he need to hike to get back down to the tram station?

(28, 43) (218, 453) (218, 493)

1. Solve (28, 43) using a model or diagram
2. Solve (28, 43) using properties of operations
3. Solve (28, 43) using place value
4. Solve (28, 43) using an “alternative” algorithm

Use Place Value Blocks

16 + 32



40 + 8

48

Count on by 10's and 1st Start with bigger

32 +10 42 +6 48

Decompose 10's and 1's

16 → 10 + 6

32 → 30 + 2

10 + 6

30 + 2

40 + 8 = 48

16 + 32

1 ten + 6 ones

3 tens + 2 ones

4 tens + 8 ones

40 + 8

48

Make a friendly number

32 + 78

32 → 30 (−2)

78 + 2 = 80

80 + 30 = 110

Strategic Decomposing

377 + 444

Connor's Thinking

377 + 444
400 40 4
3 1

377 + 3 = 380
380 + 40 = 420
420 + 400 + 1 = 821

Reese's Thinking

444
440 4
3 1

440
320 120

320
300 20

377 + 3 = 380
380 + 120 = 500
500 + 300 = 800
800 + 20 + 1 = 821

Emily's Thi

377 + 444
30 10 3

300 + 400 = 700
70 + 30 = 100
7 + 3 = 10
10 + 1 = 11
700 + 100 + 10 + 11 = 821

Big Ideas

- There's more than one way to strategically decompose numbe
- To strategically decompose, you have to analyze the numb
- Making friendly tens, hundreds, and thousands is an efficie way to solve problems.

Addition Strategies

Decomposing Add by Place Value

37 + 93

30 7 90 3

90 + 30 = 120
7 + 3 = 10
120 + 10 = 130

Make Friendly Numbers - 10's

37 + 93

90 3

37 + 3 = 40
40 + 90 = 130

Make Friendly Numbers - 100

37 + 93

30 7

93 + 7 = 100
100 + 30 = 130

Decompose One Addend and Counton

29 + 72

10 10 9

72 + 10 = 82
82 + 10 = 92
92 + 9 = 101

Make a Friendlier Addend

29 + 72 = 30 + 71

-1 +1 +1 -1

29 +1→ 30
+72 -1→ +71
101

29 -8→ 21
+72 +8→ 80
101

Make a Friendlier Addend

529 + 605

529 +1→ 530
605 -1→ 604

530 + 600 = 1,130
1,130 + 4 = 1,134

548
+433

548 + 433

500 48 400 30 3 (-1, 2)

500 + 400 = 900
900 + 48 = 948
948 + 2 = 950
950 + 30 + 1 = 981

529 + 605 =

600 4 1

500 100

529 + 500 = 1,029
1,029 + 1 = 1,030
1,030 + 100 + 4 = 1,134

529 + 605 =

500 20 4 600 5

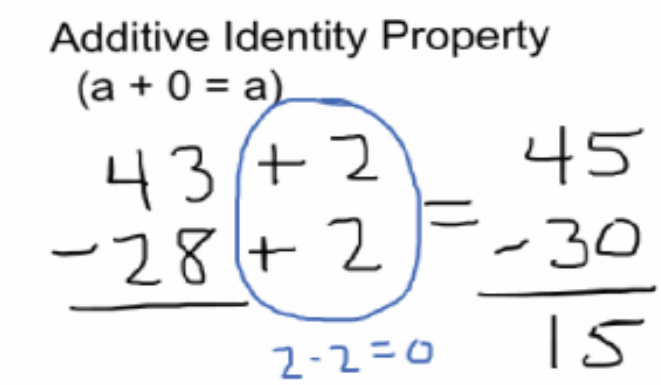
300 300 200 100

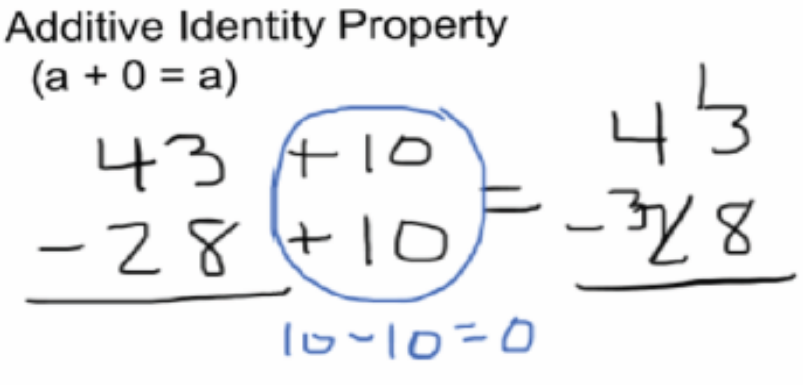
500 + 300 = 800
800 + 200 = 1,000
1,000 + 100 = 1,100
1,100 + 20 + 9 = 1,129
1,129 + 5 = 1,134

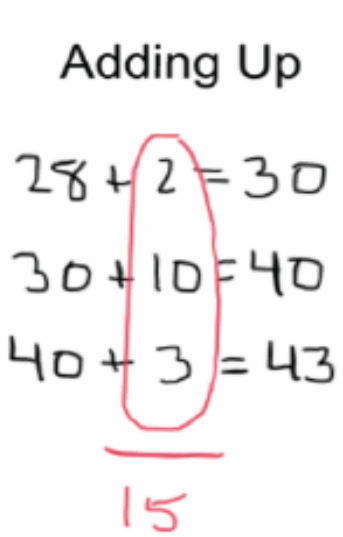
- 3rd Grade 70-34

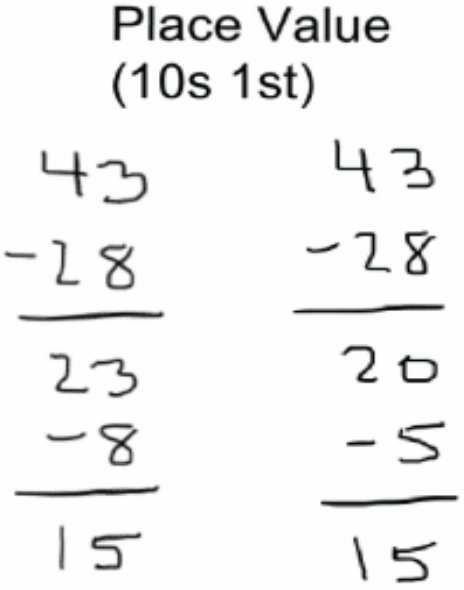
Strategies and Algorithms

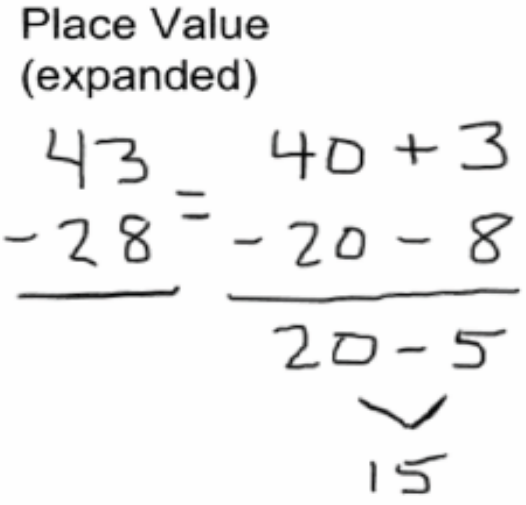
Properties of Operations, Place Value, Relationship btwn + & -



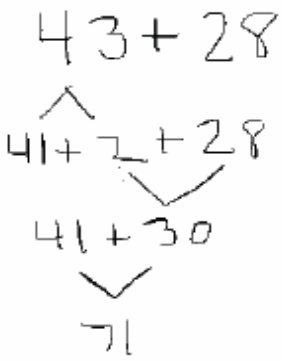


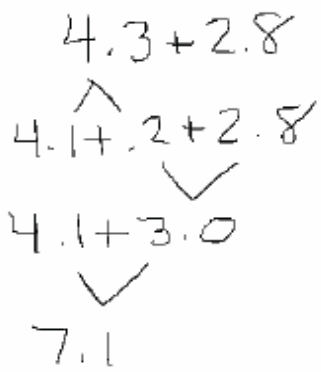


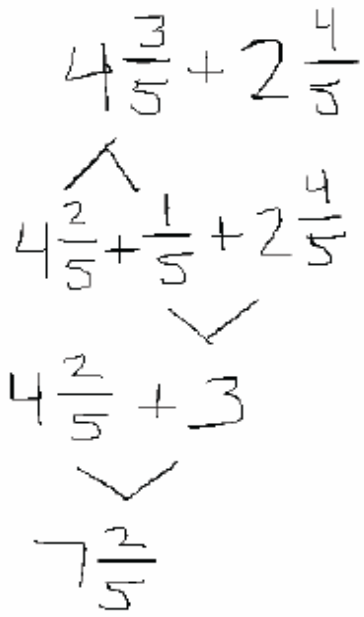


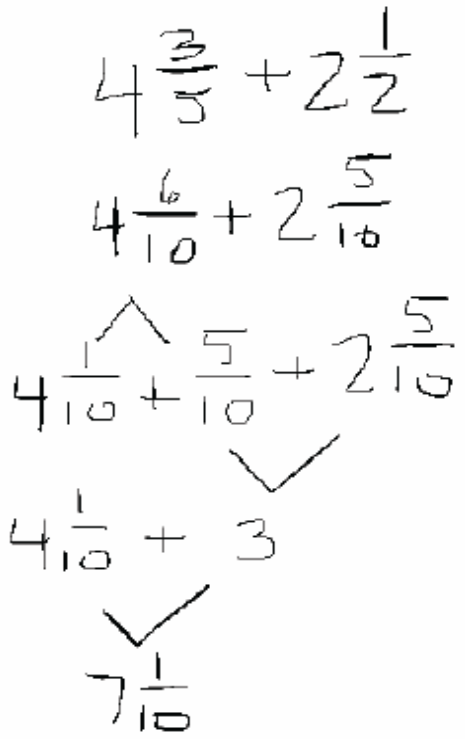


Decomposing/Recomposing (Associative Property)

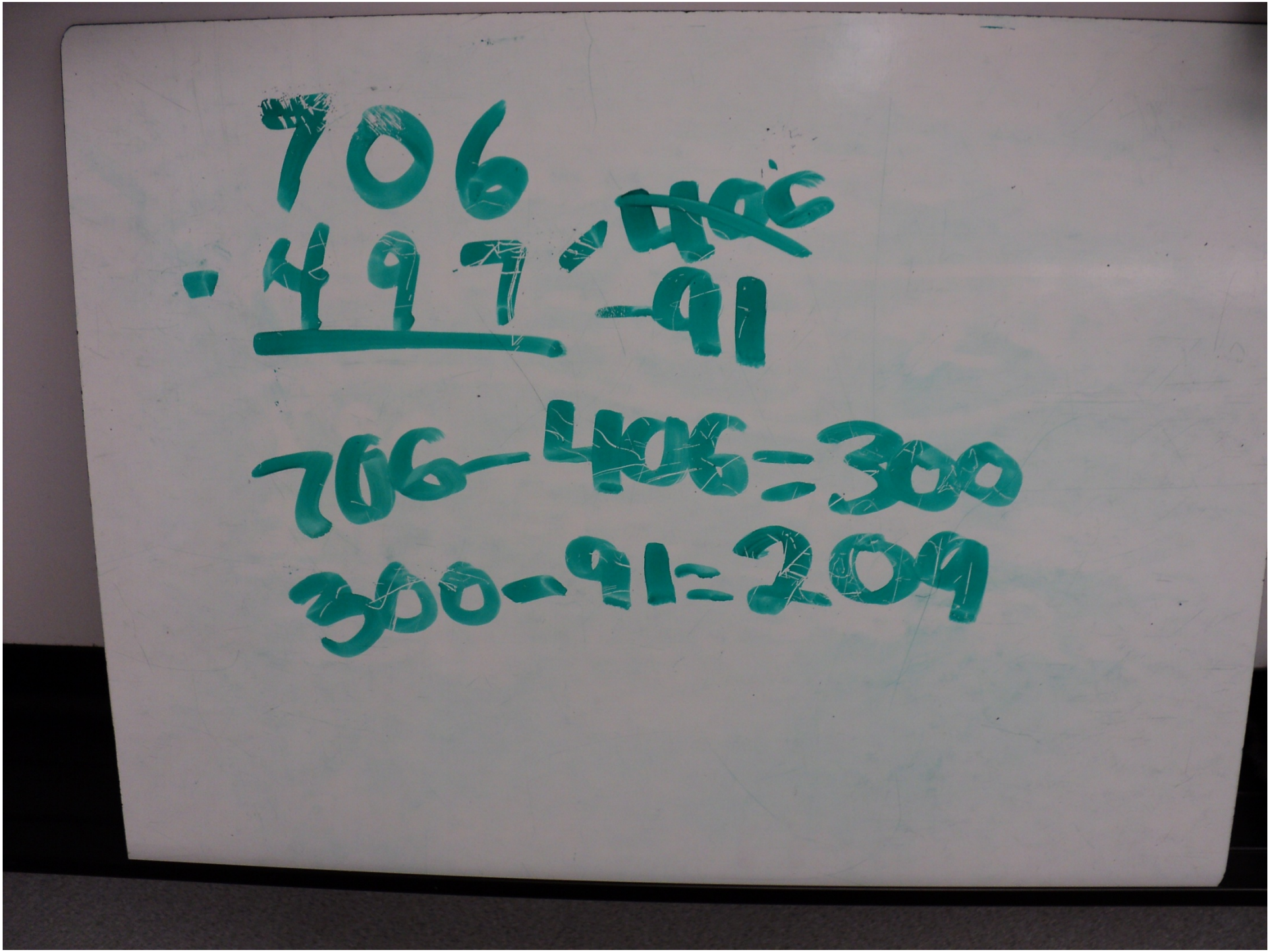








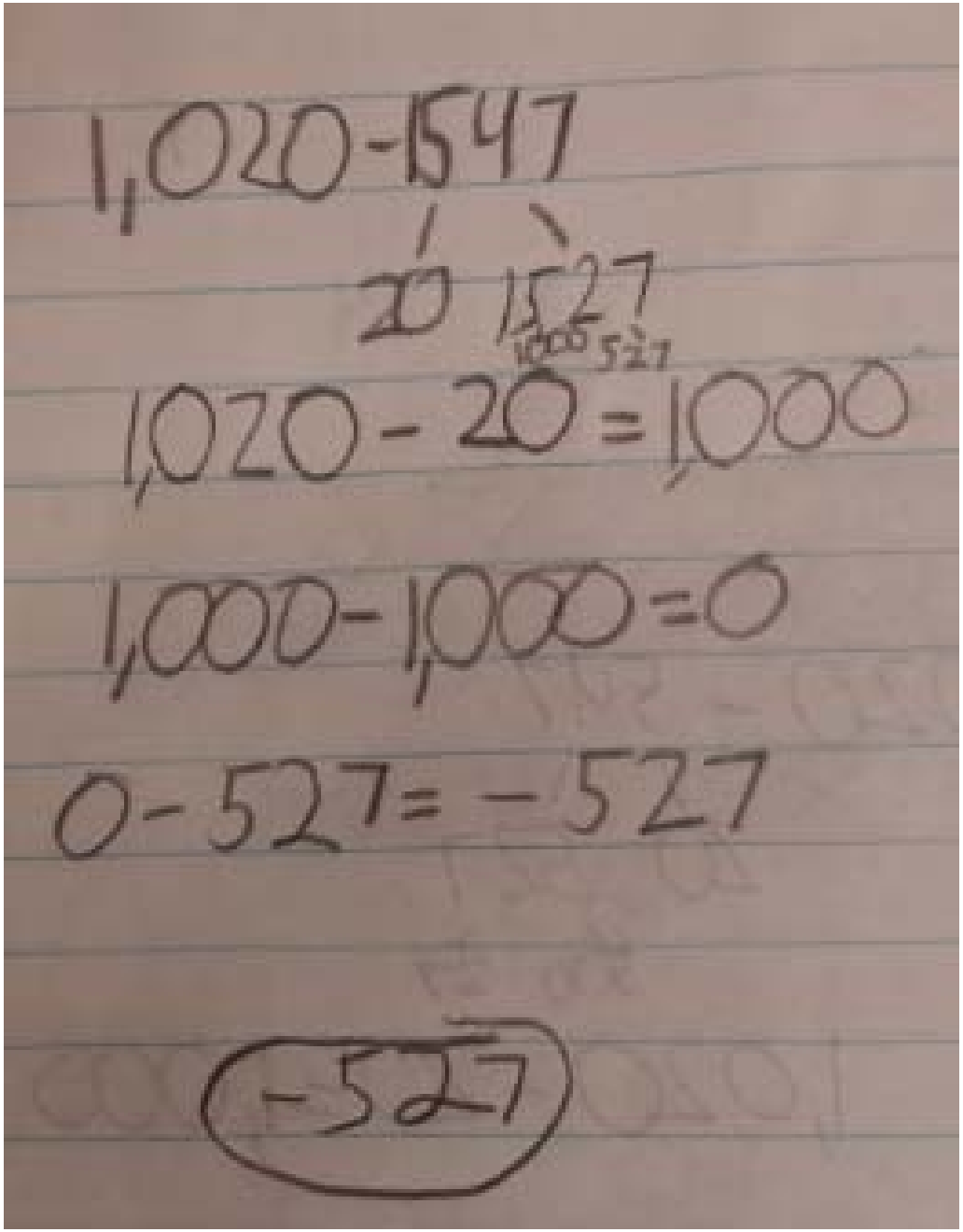
706 - 497

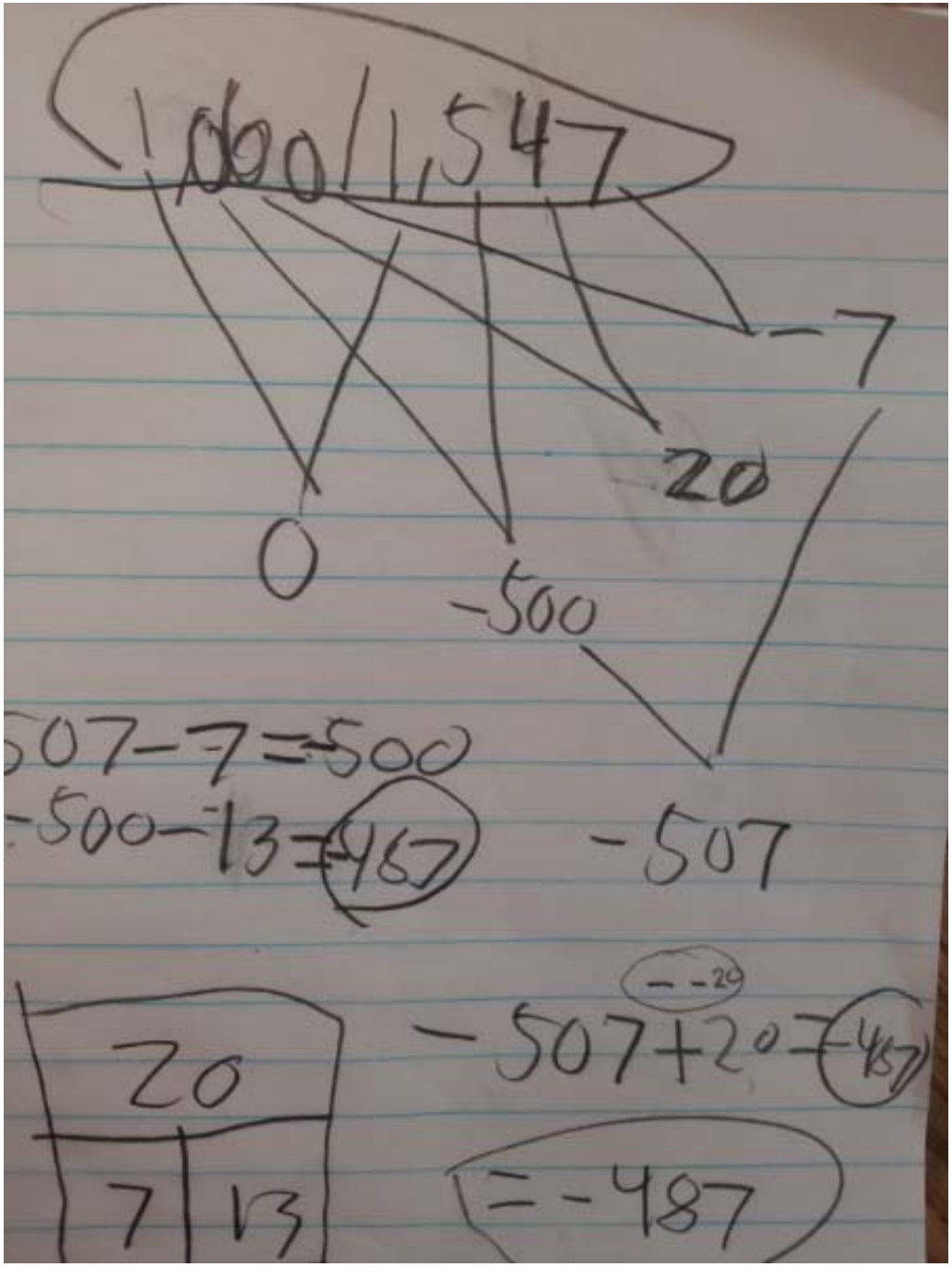

706
- 497
- 91
706 - 406 = 300
300 - 91 = 209

Caleb went on a hike in Death Valley. He took a tram up the mountain and started at an elevation of _____ feet above sea level. He hiked down in elevation _____ feet until he got back to the car. At what elevation was the car?

(90, 54) (900, 547) (1020, 547) (1020, 1547)

90-54
90-50=40
40-4=36 feet above sea level


1,020-1547
20 1527
1000 527
1,020 - 20 = 1,000
1,000-1,000=0
0-527=-527
-527


1,000/1,547
-7
20
0
-500
507-7=500
-500-13=487
-507
20
7
13
--20
-507+20=-487
=-487

Subtraction

14 - 8
↑
Minuend

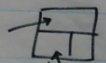
Henri Decompose the Subtrahend to get to an even 10

17 - 8
−7 ^ 7 1
10
−1
9

Gerald Count On in chunks to get to an even 10

8 →(+2) 10 →(+7) 17

Cameron Decompose minuend to make the subtrahend easier to subtract



83 - 26
53 ^ 30
−26
53 + 4 = 57

Zack Count Back in chunks

83 −3→ 80 −50→ 50 +

Andrea Slide the difference to a friendly equation

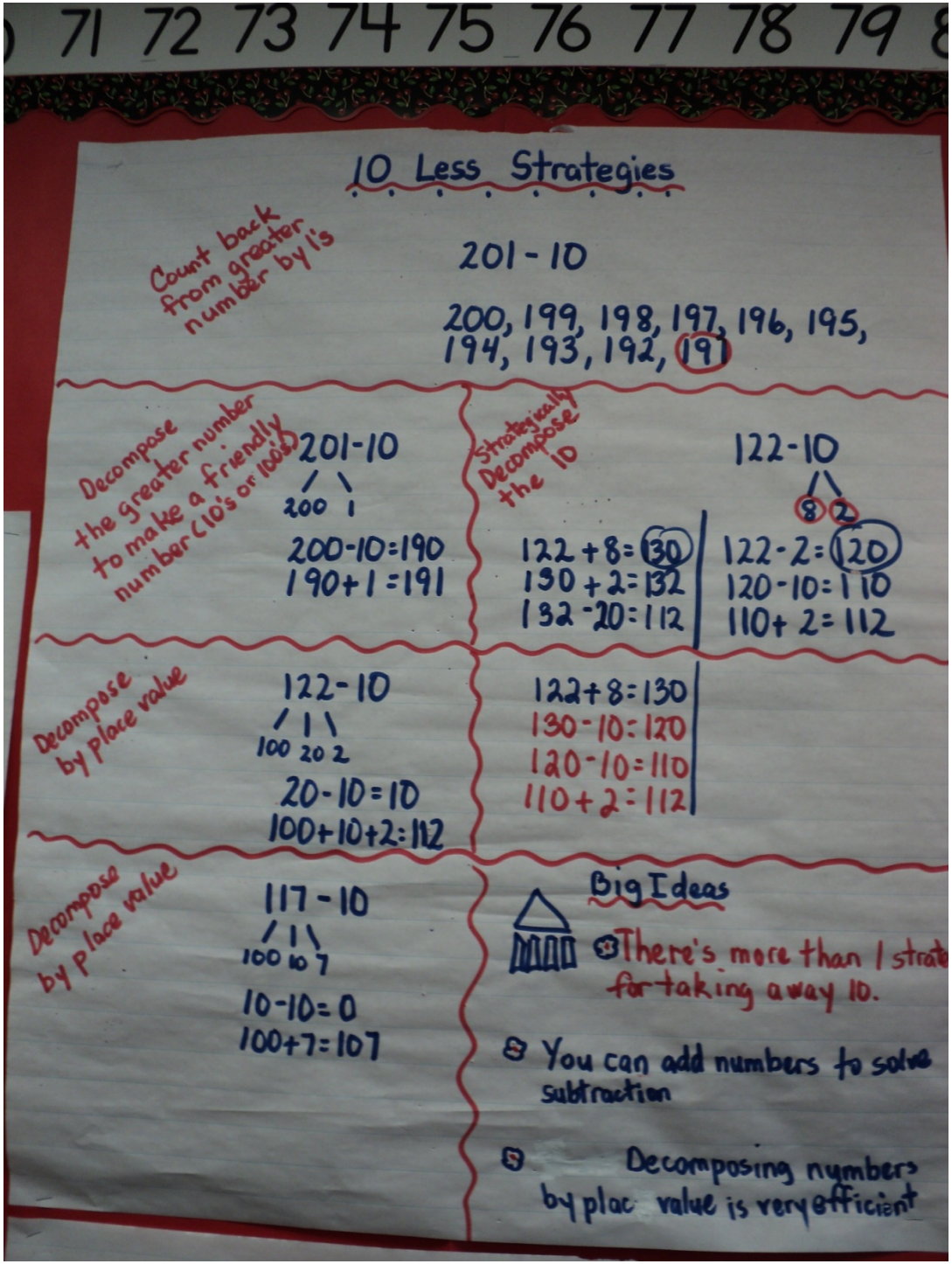
83 - 26
+4 +4
87 - 30 = 5

Faith Decompose minuend and subtrahend in friendly chunks

829 - 362
400 ^ 60 ✗ (300)
−2 −300
427 100
−60
+40 = 46

Nolan Slide the equation in 2 steps to make the #s almost the same

829 − 362
↓ −33
829 − 329 = 500
−33
46


71 72 73 74 75 76 77 78 79
10 Less Strategies
Count back from greater number by 1's
201 - 10
200, 199, 198, 197, 196, 195, 194, 193, 192, 191
Decompose the greater number to make a friendly number (10's or 100's)
201-10
200 1
200-10=190
190+1=191
Strategically Decompose the 10
122+8=130
130+2=132
132-20=112
122-10
8 2
122-2=120
120-10=110
110+2=112
Decompose by Place value
122-10
100 20 2
20-10=10
100+10+2=112
122+8=130
130-10=120
120-10=110
110+2=112
Decompose by Place value
117-10
100 10 7
10-10=0
100+7=107
Big Ideas
There's more than 1 strate for taking away 10.
You can add numbers to solve subtraction
Decomposing numbers by plac value is very efficient

ADDITION & SUBTRACTION WORD PROBLEM TYPES

(CGI wording/Common Core wording)

	Result Unknown	Change Unknown	Start Unknown
Join/ Add To	Pete had 8 apples. Ann gave Pete 5 more apples. How many apples does Pete have now? [1]	Robin had 5 toy cars. Her parents gave her some more toy cars for her birthday. Then she had 11 toy cars. How many toy cars did her parents give Robin for her birthday? [2]	Bob had some cookies in his lunch bag and then got 2 more cookies from a friend. Now he has 8 cookies. How many cookies did Bob have in the beginning? [9]
Separate/ Take From	Tracy had 12 marbles. Then she gave 3 marbles to Fama. How many marbles does Tracy have now? [3]	Jesse had 11 pieces of candy. He lost some of the pieces. Now he has 4 pieces of candy. How many pieces of candy did Jesse lose? [6]	Colleen had some guppies. She gave 3 guppies to Roger. Then she had 5 guppies left. How many guppies did Colleen have to start with? [10]

	Total Unknown	Addend Unknown	Both Addends Unknown
Part-Part-Whole/Put Together-Take Apart	6 boys and 4 girls were playing soccer. How many children were playing soccer? [4]	Sever and Becky have 8 books when they put all their books together. Sever has 3 books. How many books does Becky have? [11]	There are 10 children playing soccer. How many can be boys and how many can be girls? [?]

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	Janet has 3 balloons. Her sister Connie has 9 balloons. How many more balloons does Connie have than Janet? [how many more; 5]	Luis has 6 pet fish. Carla has 2 more fish than Luis. How many fish does Carla have? ["more" suggests operation; 12]	Joy has 7 mice. She has 4 more mice than Mark. How many mice does Mark have? ["more" suggests wrong operation; 13]
	Janet has 3 balloons. Her sister Connie has 9 balloons. How many fewer balloons does Janet have than Connie? [how many fewer; ?]	Luis has 2 fewer fish than Carla. Luis has 6 pet fish. How many fish does Carla have? ["fewer" version suggests wrong operation; ?]	Mark has 4 fewer mice than Joy. Joy has 7 mice. How many mice does Mark have? ["fewer" suggests operation; ?]

Claim 3: Communicating Reasoning

<table>
<tr><th>Method W</th><th>Method Z</th></tr>
<tr><td>

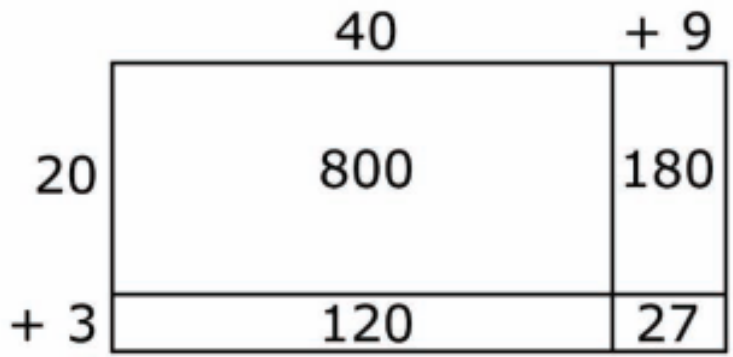
23 × 49

20 × 9 = 180
3 × 9 = 27
20 × 4 = 80
3 × 4 = + 12
299

</td><td>

23 × 49

Area Model



Rectangle Sections

1
800
120
180
+ 27
1,127

</td></tr>
</table>

Identify the method where Pablo made a mistake and explain what he should do to correct it.

Sample Top-Score Response:

Pablo made a mistake when using Method W. He should have multiplied 20 and 3 by 40 instead of by 4. He made a place-value error. Multiplying by 40 instead of by 4 would have resulted in the same answer as when he used Method Z (1,127).

5th Grade

5. Fluently multiply multi-digit whole numbers using the standard algorithm.

4th Grade

5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Associative Property of Multiplication

- 3rd Grade (7:20-8:40)
- Also (10:08-14:10)

-18 -17 -16 -15 -14 -13 -12 -11 -10 -9

Norms for Sharing

- Listen to people sharing their thinking
- Learn from other people's thinking
- Understand /Ask questions to help you understand
- Give respect to person talking
- Look at the person who is talking ~ pay attention

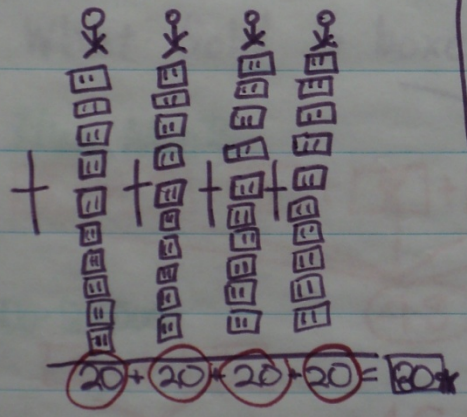
Associative Property

- 5^{th} Gr 15 x 12

Associative Property of #1 Multiplication

Triple

Four children made 10 Valentine cards each. They put 2 candy hearts in each card. How many candy hearts are in all the Valentine cards?

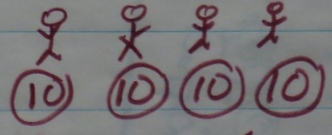


$4 \times (10 \times 2)$

$4 \times 10 = 40$

$\times 2$

80



40 cards $\times 2 = 80$

$(4 \times 10) \times 2$

10 $\times 2$ 10 $\times 2$ 10 $\times 2$ 10 $\times 2$

Identity Property of Multiplication — Any number x 1 is equal to that # or 1 x

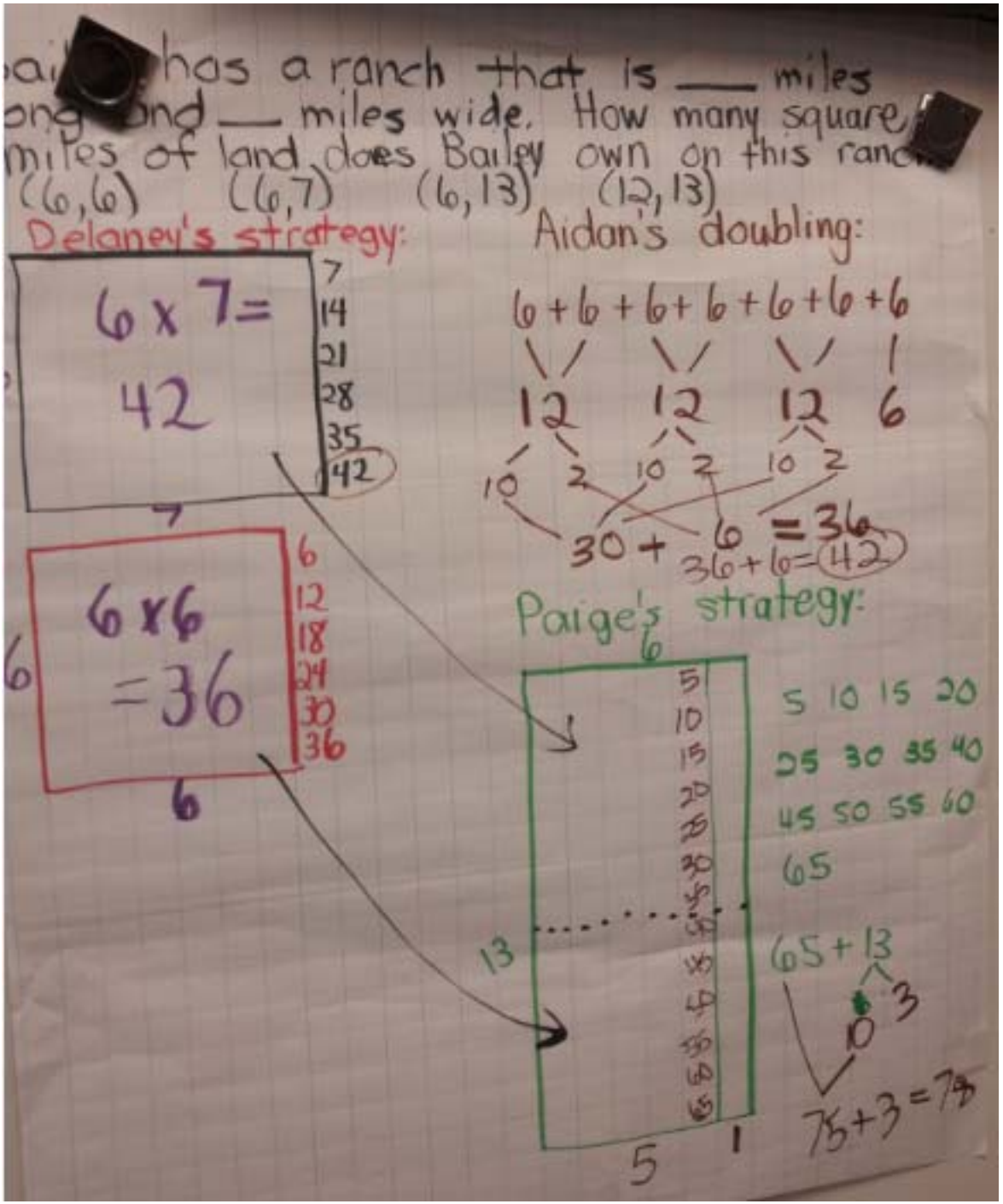
Commutative Property of Multiplication — If you switch the order of the factors, then you get the same product (total).

Zero Property of Multiplication — Any # times 0 has a product of 0

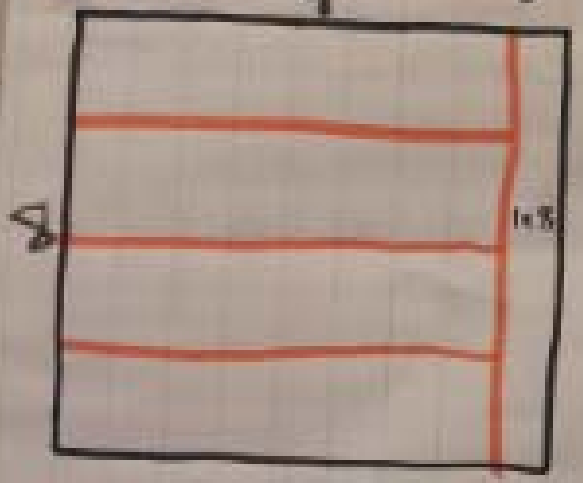
Associative Property of Multiplication — You can group factors in any way and get the same product

$(a \times b) \times c = a \times (b \times c)$

Distributive Property of Multiplication — You can break up the groups into parts and multiply the parts by their amounts and add those groups.


has a ranch that is ___ miles
ong and ___ miles wide. How many square
miles of land does Bailey own on this ranch
(6,6) (6,7) (6,13) (12,13)
Delaney's strategy:
6 x 7 =
42
7
14
21
28
35
42
7
6 x 6
=36
6
12
18
24
30
36
6
6
Aidan's doubling:
6+6+6+6+6+6+6
12 12 12 6
10 2 10 2 10 2
30 + 6 =36
36+6=42
Paige's strategy:
6
13
5
1
5 10 15 20
25 30 35 40
45 50 55 60
65
65+13
10 3
75+3=78

Kylee's doubling: (complex)



16 16 16 16

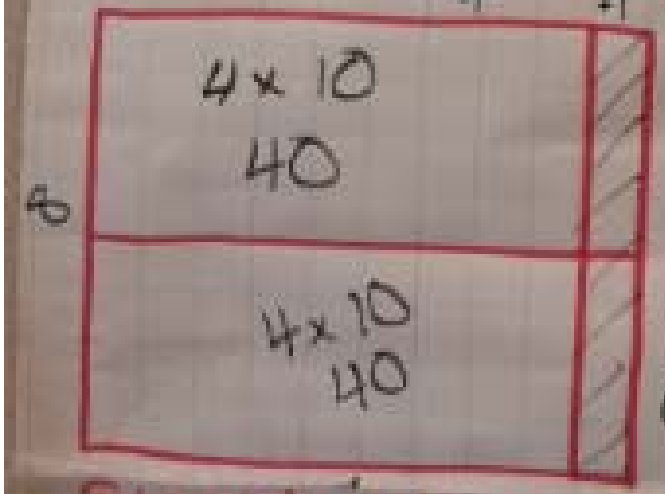
16 + 16 = 32

32 + 32 = 64

64 + 8 = 72

How many 9s?
4
8
9

Abby's compensating:



80 - 4

9 x 8 =

(10 - 1) x

(10 x 4) +

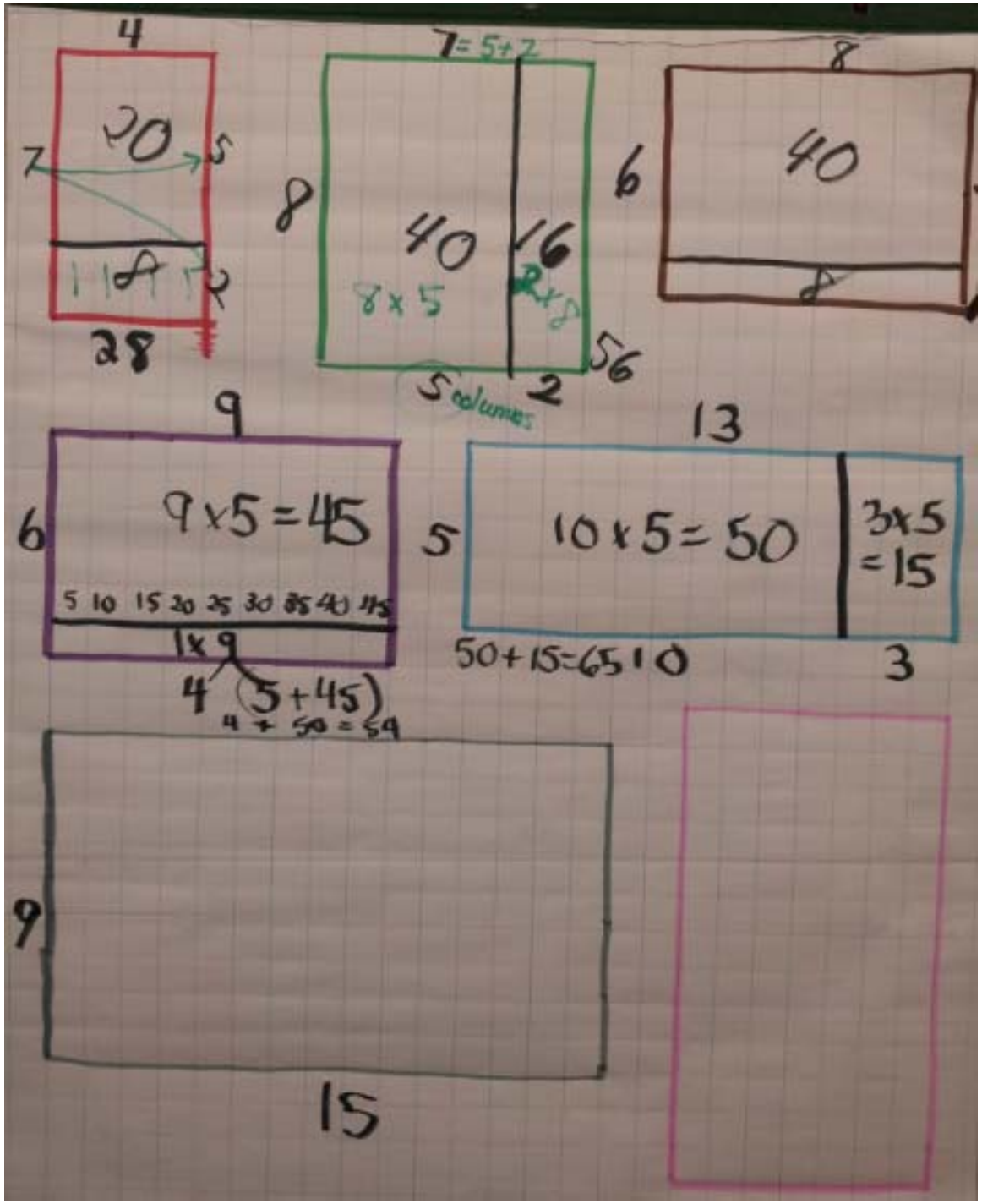
40

Gianna's thinking:

Conjectures for

Even Num

- When you add 2 even
the sum will be even.
- If I start at an even n
count by an even number
will be even. even+even=e
- Even numbers can be


4
20
5
7
28
7=5+2
8
40
16
8x5
2x8
56
5 columns
2
8
6
40
8
9
6
9x5=45
5 10 15 20 25 30 35 40 45
1x9
4
5+45)
4+50=54
13
5
10x5=50
3x5
=15
50+15=6510
3
9
15

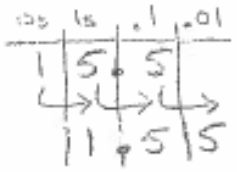
Associative Property

$3.1 \times .5$

$3.1 \times 5 \times .1$

$15.5 \times .1$

1.55



Associative Property

$3.1 \times .5$

$3.1 \times .1 \times 5$

$.31 \times 5$

1.55

$\frac{31}{100} \times 5 = \frac{155}{100}$

$= 1.55$

Mult. Inverse & Mult. Identity Prop.

$3.1 \times .5$

$\div 2 \quad \times 2$

$\frac{3.1}{2} \times 1$

$\frac{3}{2} + \frac{.1}{2}$

$1.5 + .05$

1.55

$3.1 \times .5 \times \frac{2}{2}$

Mult. Inverse & Mult. Identity Prop.

$3.1 \times .5$

$\div 10 \quad \times 10$

$.31 \times 5$

$.31 + .31 + .31 + .31 + .31$

$.93 + .62$

$.93 + .07 + .55$

$1.00 + .55$

1.55

$3.1 \times .5 \times \frac{10}{10}$

Fraction-Decimal Connection

$3.1 \times .5$

$3\frac{1}{10} \times \frac{5}{10}$

$(3 + \frac{1}{10}) \cdot \frac{5}{10}$

$3(\frac{5}{10}) + \frac{1}{10}(\frac{5}{10})$

$\frac{15}{10} + \frac{5}{100}$

$\frac{150}{100} + \frac{5}{100}$

$\frac{155}{100} = 1.55$

Distributive Property

$3.1 \times .5$

$(3 + .1) \times .5$

$3(.5) + .1(.5)$

$1.5 + .05$

1.55

Place Value (partial products)

3.1

$\times .5$

1.5

$.05$

1.55

WN x Alg. (estimate dec.)

3.1

$\times .5$

1.55

$3 > 3 \times \frac{1}{2} > 1$

Standard Algorithm

3.1

$\times .5$

1.55

6th Grade

2. Fluently divide multi-digit numbers using the standard algorithm.

5th Grade

5.NBT.6 Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

4th Grade

6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

- 5th Gr Number Talk
  - 300 / 15

Word Problem Types: Multiplication/Division (K-5 OA p. 23)

Problem Type	Unknown Product	Group Size Unknown	Number of Groups Unknown
Equal groups of objects 3rd Grade	5 taxis have 4 passengers in each. How many passengers are there altogether?	20 passengers have to fit equally into 5 taxis. How many passengers go in each taxi?	20 passengers travel 4 to a taxi. How many taxis are needed?

Problem Type	Unknown Product	Unknown Factor	Unknown Factor
Arrays of Objects 3rd Grade	There are 5 rows of pictures with 4 pictures in each row. How many pictures are there? The pictures on the wall are arranged in 5 rows and 4 columns. How many pictures are there?	20 pictures are arranged into 4 equal rows. How many pictures will be in each row? 20 pictures are arranged in 4 rows. How many columns of pictures are there?	20 pictures are arranged into rows with 5 pictures in each row. How many rows will there be? 20 pictures are arranged into 5 columns. How may rows are there?
Rectangular Area* 3rd Grade	A rectangle has sides of length 8 and 7 units. What is the area of the rectangle?	A rectangle has an area of 56 square units. One side is 8 units long. What is the length of the other side?	

Multiplicative Comparison 4th Grade	Henry has six times as many marbles as Sarah. Sarah has 12 marbles. How many marbles does Henry have? (larger unknown)	Henry has six times as many marbles as Sarah. Henry has 72 marbles, how many does Sarah have. (smaller unknown)	Henry has 72 marbles and Sarah has 12 marbles. How many times more marbles does Henry have than Sarah? (multiplier unknown)
5th Grade	Henry has 72 marbles. Sarah has one-sixth as many marbles as Henry. How many marbles does Sarah have? (smaller unknown; comp. factor < 1)	Sarah has 12 marbles. That is one-sixth as many marbles as Henry has. How many marbles does Henry have? (larger unknown; comp. factor <1)	Henry has 72 marbles and Sarah has 12 marbles. What fraction of Henry's quantity of marbles does Sarah have? (multiplier unknown; comp. factor <1)

Thanks for all the work you do with students!

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